BALL / CYLPEBS SIZING FOR BALL MILLS

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January 2024 / Turkey

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1 . Introduction

The weight of the cast ball or cylpebs to be used in grinding should be such that when it freely falls inside the mill, it should be able to break the largest size of ore in the mill in terms of diameter or weight.

It is essential to know the *density* of the mill when loading, as well as the *bulk density*. While the density of the material from which the mill is produced remains constant, the bulk density depends on the size of the mill and changes as it wears inside the mill.

Density is the ratio of the mass (Q) of the material at 20°C ambient temperature to its volume (V).

Density is measured is as follows:

- *Water is filled into a clean and completely dry calibrated glass tube with a volume of 1 dm^3, marked with water levels in cm^3 equivalent, up to a certain level "h1," and its weight "Q_1" is measured on a precision scale with a sensitivity of 0.1 mg.*
- *The material which density is to be measured is gently placed into the calibrated glass tube without creating air bubbles.*
- *The rising water level "h^2" is determined by reading. The volume of water rising in the calibrated glass tube is equal to the volume of the material which density is being measured.*
- *The calibrated glass tube is weighed again to determine the weight "Q_2." The difference between the two weighings is equal to the weight of the material inside the calibrated glass tube.*

Following these measurements, the material density can be calculated as follows:

$\rho = (Q_2 - Q_1) / (h_2 - h_1)$ gr/cm³

Where;

Q_1 and Q_2 are the results of the first and second weighings,
h_1 and h_2 are the first and second water levels, respectively.

By using this formula, the average of the three repetitions can be calculated. If this average shows a deviation of less than 0.005 gr/cm^3 at 20°C, the obtained density measurement can be considered acceptable.

The density measurement is illustrated in Figure 1.

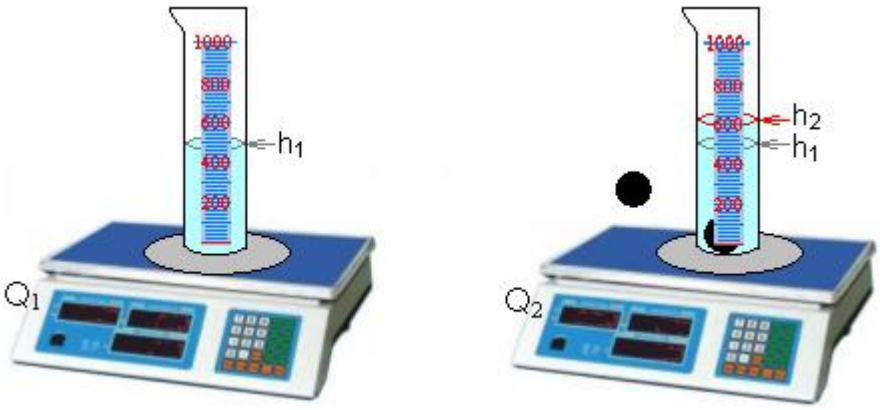

Image 1: Density Measurement

For example, when the difference between the two weighings is 450 grams, and the corresponding water level difference is 135 cm³, the material density (ρ) can be calculated as follows:

$\rho = (Q_2 - Q_1 = 450 \text{ gr}) / (h_2 - h_1 = 135 \text{ cm}^3) = 3.33$ gr/cm³

So, in this example, the material density would be 3.33 grams per cubic centimeter.

The weight of the grinding material in bulk, expressed as a ratio to its volume, is referred to as *bulk density.*

When measuring bulk density, a vessel which volume is 1.0 m³ or 0.50 m³, is filled with balls/cylpebs or gravel, and its weight is measured. The ratio of the measured weight to the vessel provides the bulk density of the material. Alternatively, the option exists to fill the vessel containing balls/cylpebs or gravel with water to measure the void space between the balls/cylpebs and gravel. Determining bulk density is crucial for establishing the amount of balls/cylpebs and gravel to be loaded into the mill and the pulp volume between them.

Table 1 provides the density and bulk density of various materials used in grinding mills.

Table 1: Density and Bulk Density of Grinding Media

Grinding media	Gap in the granding media, %	Density t/m^3	Bulk density t / m^3
Casting balls	40	7.80	4.50
Casting cylpebs	35 - 38	7.80	4.90
Alumina/ceramic balls	40	3.20-3.90	2.20
Silica- pebbles	40	2.65	1.7-1.90

2.Ball Diameter Calculation

In ball mills, the *"operating ball diameter"* or size of the steel or cast iron balls to be used is calculated using the Bond equation (Bond, 1961).

$$Ø_B = 25.4 * (F_{80} / K_D)^{0.5} * \{(\rho_C * W_i) / [100 * N_D (3.281 * Ø_D)^{0.5}]\}^{0.33}$$

$Ø_B$ = Operating ball diameter, mm

F_{80} = Size at which 80% of the material passes, µ

W_i = Work index, kWs/t

N_D = Mill speed as a percentage of critical speed

ρ_C = Density of the ore to be ground, gr/cm³

$Ø_{Diç}$ = Mill inside diameter, m

K_D = Correction factor, to be taken from Table 2

Çizelge 2: Correction factor, K_D

Mill type	Wet/Dry grinding	Grinding circuit	K_D value
Overflow	Wet grinding	Open circuit	350
Overflow	Wet grinding	Closed circuit	350
Diaphragm	Wet grinding	Open circuit	330
Diaphragm	Wet grinding	Closed circuit	330
Diaphragm	Dry grinding	Open circuit	335
Diaphragm	Dry grinding	Closed circuit	335

Large-sized mills and ball diameters are used in the grinding of coarse ore particles. When Ø < 60 mm balls are required, mills with two lifters are recommended, and when Ø > 60 mm balls are used, mills with a single lifter are suggested.

For Ø < 20 mm, alumina balls are specially manufactured. When Ø = 20 mm balls are loaded into the mill for fine grinding, the balls wear down, creating an appropriate distribution with a -20 mm size during the grinding process.

The cost of smaller-sized balls is higher depending on the ball diameter. Larger mills consume more energy with smaller diameter balls.

Grinding process is influenced by the weight of the ball depending on its diameter and the amount of balls loaded into the mill. The calculated weight of cast balls based on their diameter is fundamental in calculations for mill size, ball diameter, and the amount of grinding material to be fed into the mill.

The density of the ball varies according to the material it is made of. Table 3 provides the weight and surface areas based on the sizes of balls made of different materials. In this table, the density of the material from which the ball is made is assumed to be 7.85 gr/cm³.

Table 3: Physical Properties of the Ball (Doering International)

Diameter, $\varnothing_B$, mm	Volume cm^3/ ball	Weight ρ_B= 7.85, gr/ball	Weight kg/m^3	Approximate ball pieces ρ_B=7.8, ball/ton	Surface area cm^2/bilya	Specific surface area, cm^2/gr
100	523.6	4087.0	4560	244	314.0	0.07
90	381.7	2977.0	4590	335	254.0	0.08
80	268.0	2091.0	4620	478	201.0	0.09
70	179.6	1401.0	4640	713	154.0	0.10
60	113.1	882.0	4660	1.133	113.0	0.12
50	65.5	511.0	4708	1.956	78.0	0.15
45	47.7	372.0	4500	2.688	63.6	0.17
40	33.5	264.0	4500	3.788	50.0	0.19
35	22.5	175.0	4500	5.714	38.5	0.22
30	14.0	112.0	4500	8.928	28.0	0.25
25	8.2	65.0	4500	15.384	20.0	0.31
20	4.2	33.0	4500	30.303	12.5	0.38
15	1.8	13.0	4500	76.923	7.0	0.54
10	0.5	4.5	4500	222.222	3.1	0.69

3.Calculation of Cylpebs Dimensions

After calculating the ball diameter, it is necessary for cylpebs, which can perform the same grinding, to have the same weight as well:

$Q_{BİLYA} = Q_{SİLPEBS}$

$4/3 * \pi * \phi_B^3 * \rho_D = \pi * \phi_S^2 * h * \rho_D$

$4/3 * \phi_B^3 = \pi * \phi_S^2 * h$

The height of cylpebs is equal to its diameter.

$4/3 * \phi_B^3 = \phi_S^2 * (2 * \phi_S)$

$\phi_S = 0{,}8735\ \phi_S$

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After calculating the ball diameter, the diameter and height of cylpebs are determined by taking 0.8735 times the ball diameter. *As a good approximation, when cylpebs are desired to be used instead of balls, the calculated ball diameter is typically multiplied by approximately 0.90 to determine the diameter and height of cylpebs.*

The height of cylpebs used as a grinding medium is generally equal to its diameter. In mass production, when it is desired for easy removal of cylpebs from the mold, the upper and lower cylindrical diameters of cylpebs molds may be prepared with slight differences.

The required sizes of grinding balls and cylpebs are calculated by the following equations, as determined by *DOERING International GmbH*.

$$Ø_B = 20.17 * (F_{80} / K_D)^{0.5} * \{(\rho_C * W_i) / [N_D (Ø_{Diç})^{0.5}] \}^{0.33}$$ The ball diameter

$$Ø_S = 18.15 * (F_{80} / K_D)^{0.5} * \{(\rho_C * W_i) / [N_D (Ø_{Diç})^{0.5}] \}^{0.33}$$ The cylpebs size

Table 4 provides the weight and surface areas of cylpebs based on their sizes. In this table, the density of the material from which the cylpebs are made is assumed to be 7.67 gr/cm^3.

Table 4: Physical properties of cylpebs (Doering)

Cylpebs ∅S* h, mm	Weight ρS= 7.67, gr/cylpebs	Surface area cm^2/cylpebs	Specific surface area, cm^2/gr	Bulk density t /m^3
54*54	880.0	137.4	0.15	4.9
45*45	560.0	95.4	0.17	4.9
40*40	379.0	75.4	0.20	4.9
35*35	245.0	57.7	0.24	4.9
32*32	190.0	48.3	0.25	4.9
28*28	128.0	37.0	0.28	4.9
24*24	82.5	27.0	0.33	4.9
22*22	62.0	22.8	0.37	4.9
20*20	45.0	19.0	0.40	4.9
16*16	23.5	12.0	0.51	4.9
12*12	11.1	6.8	0.55	4.9
10*10	6.3	4.7	0.75	4.9
6 * 8	3.0	3.0	1.00	4.9

4.Alumina and Pebble Ball Size Calculation

The diameters of steel or cast iron balls made from a material with a density of 7.8 gr/cm³ will not be the same as those made from a lower-density material that can be used in the same grinding process. As the density of the material from which the ball is made decreases, the required ball diameter for grinding will increase.

Therefore, the first step is to calculate the cast ball diameter that can grind the raw material in the mill. Then, the size of the alumina ball or pebble is determined to be equal to the calculated cast ball weight.

$Q_{XB} = Q_{DB}$

Q_{XB} = Weight of low-density ball

Q_{DB} = Weight of cast ball

$$4/3 * \pi * \phi_X^3 * \rho_X = 4/3 * \pi * \phi_D^3 * \rho_D$$

$$2\,d_X = 2 * d_D * (7.8 / \rho_X)^{0.33}$$

$$Ø_X = Ø_D * (7.8 / \rho_X)^{0.33}$$

ϕ_X = Radius of low-density ball, mm

ρ_X = Density of the ball, gr/cm^3

ϕ_D = Radius of cast ball, mm

ρ_D = Density of cast ball, 7.8 gr/cm^3

$Ø_X$ = Diameter of low-density ball, mm

$Ø_D$ = Diameter of cast ball, mm

For example; for grinding quartz with a Wi (work index) of 16 kWs/t, a density of 2.7 gr/cm^3, and F80 of 2,000µ, the required steel ball diameter is 37.5 mm. If cylpebs are to be used instead of balls, the calculated cylpebs diameter and height (x 0.8735 =) will be 32.7 mm. When it is desired to grind the same quartz with alumina balls, which has a density of 3.7 gr/cm^3, the diameter of alumina balls;

$Ø_X = Ø_D * (7.8 / \rho_X)^{0.33} = 37.5 * (7.8 / 3.7)^{0.33} = 48$ mm

To be able to grind this material with quartz gravel of density 2.7:

$Ø_X = Ø_D * (7.8 / \rho_X)^{0.33} = 37.5 * (7.8/2.7)^{0.33} = 53.4$ mm gravel of diameter will be required.

The weight of a single cast ball, Q_B; $4/3 * \pi * \phi_D^3 * \rho_D = 4/3 * \pi * (37.5 / 2)^3 * 7.8 = 215$ gr

The weight of a single cast cylpebs, Q_B; $\pi * \phi_D^2 * \rho_D = \pi * (32.7 / 2)^2 * 32.7 * 7.8 = 215$ gr

The weight of a single alumina ball, Q_A; $4/3 * \pi * \phi_A^3 * \rho_A = 4/3 * \pi * (48 / 2)^3 * 3.7 = 215$ gr

The weight of a single gravel, $Q_Ç$; $4/3 * \pi * \phi_X^3 * \rho_X = 4/3 * \pi * (53.4 / 2)^3 * 2.7 = 215$ gr

Under appropriate grinding conditions, it has been experienced and observed that a "1-inch" steel ball can grind silica ore, where 80% of the ore is below "1 mm". Theoretical and experimental determinations indicate that a "2-inch" steel ball can grind a "4 mm" ore, and a "3-inch" steel ball can grind a "9 mm" ore. From these observations, a connection can be established that the "ball size in inches is equal to the square root of the ore size expressed in millimeters.

The calculated operating ball size is selected as the closest ball size available in the market. If the required ball size is less than 25 mm, the operating ball size must be chosen upper size than required ball as the closest ball size available in the market.

Regarding grinding:

- *The cost of small-sized balls is higher.*
- *Small-diameter balls pose a higher risk of plugging partition openings in compartment mills.*
- *Larger mills consume more energy with smaller diameter balls.*

5.Initial charge of ball mills

After calculating the size of the balls/cylpebs to be used in grinding, when the mill is first loaded, balls should be loaded in smaller and varying proportions than the operational size determined using the values in Table 5 to ensure the required size distribution in the grinding medium. This is referred to as the *"initial loading."*

Table 5: Distribution of balls/cylpebs in the initial loading of ball mills

Operating balls/cylpebs diameter, $\varnothing_B$, mm	Weight-based initial loading ratio (%)						
	115 mm	100 mm	90 mm	75 mm	65 mm	50 mm	40 mm
115	23.0	-	-	-	-	-	-
100	31.0	23.0	-	-	-	-	-
90	18.0	34.0	24.0	-	-	-	-
75	15.0	21.0	38.0	31.0	-	-	-
65	7.0	12.0	20.5	39.0	34.0	-	-
50	3.8	6.5	11.5	19.0	43.0	40.0	-
40	1.7	2.5	4.5	8.0	17.0	45.0	51.0
25	0.5	1.0	1.5	3.0	6.0	15.0	49.0
Toplam(%)	100	100	100	100	100	100	100

For example, when the ball diameter $\varnothing_B$ = 100 mm is calculated, the weight-wise initial ball loading of the mill would be as follows:

$\varnothing_B$ = 100 mm, %23
$\varnothing_B$ = 90 mm, %34
$\varnothing_B$ = 75 mm, %2
$\varnothing_B$ = 65 mm, %12
$\varnothing_B$ = 50 mm, %6.5
$\varnothing_B$ = 40 mm, %2.5
$\varnothing_B$ = 25 mm, %1

In this case, the initial loading of the mill will be as shown in Figure 1.

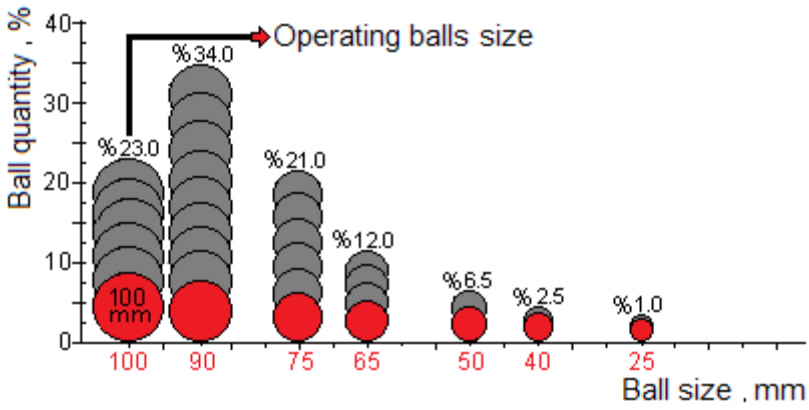


Figure 1: Initial loading of the mill when the ball diameter is calculated as 100 mm

6. Addition of grinding material to the mill after the initial loading

If the mineral processing engineer lacks the necessary knowledge for the profession, they will not inquire, will not abandon their habits even if they are incorrect, or will adapt to the habits in the facilities where they start their duties. The most significant example of this is the addition of balls/cylpebs of different diameters to the mill after the initial loading, while the mill is in operation, just as in the initial loading.

After the mill is initially loaded, the balls/cylpebs inside will wear down, reducing in size towards a lower dimension. Following this reduction, within the mill, there will be only one ball/cylpebs with the largest operational size, as seen in Figure 2, either due to a decrease in quantity or wear.

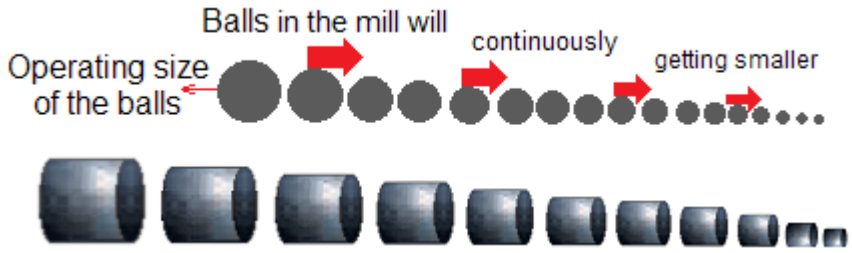


Figure 2: Reduction in size of balls/cylpebs during the grinding process in the mill

For this reason, after the initial loading, operational size balls should be continuously added to the mill as depicted in Figure 3.

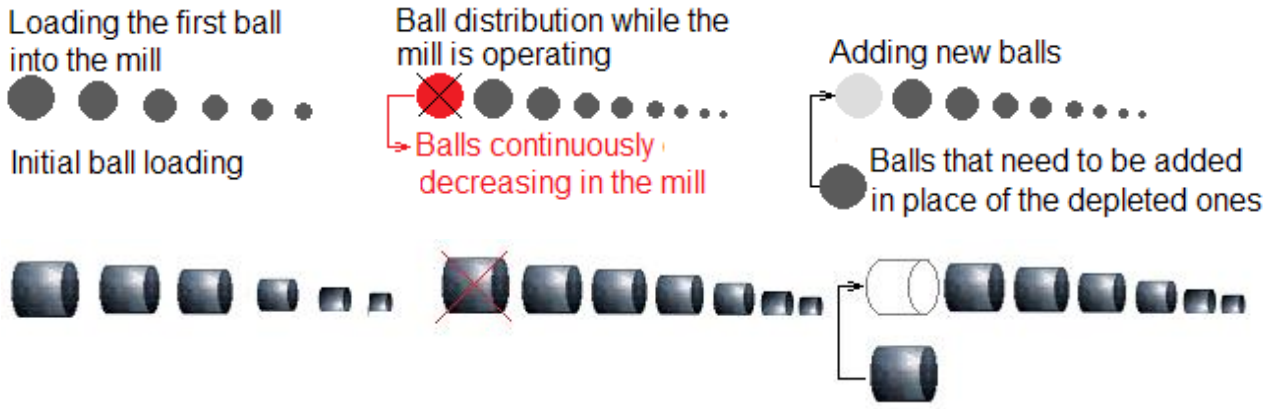


Figure 3: Loading of balls/cylpebs while the mill is in operation

When the ball diameter $Ø_B$ is calculated as 100 mm, the ball distribution in the mill while it is in operation will be as shown in Figure 4. As the balls in the mill continuously reduce in size, the only ball diminishing in size is the one with a diameter of $Ø_B$ = 100 mm, which should be added to the mill.

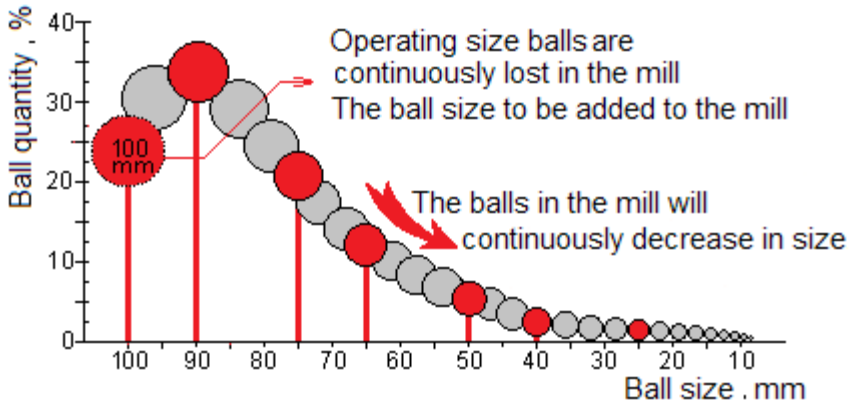


Figure 4: Lost and required addition of balls/cylpebs while the mill is in operation

As can be understood from the figure, to maintain the ball distribution from the initial loading, the ball with the calculated largest diameter should be added to the mill while it is in operation. This ensures a continuous grinding medium consistency equivalent to the size distribution in the initial loading during the grinding process inside the mill.

If the grinding process is well controlled, the required balls for the mill should be continuously loaded individually with the ore using feeders designed for this purpose, according to the ore consumption during grinding, instead of adding several tons of balls at once.

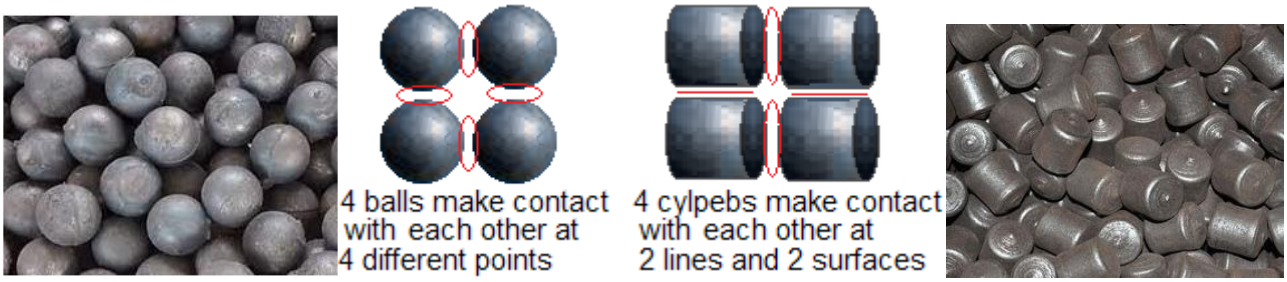
The input and output sizes of the mill are often the same in diameter. The only difference is that the spirals are inward at both throats. To reduce the outlet mouth diameter and raise the pulp overflow level, a discharge trunnion, generally made of two parts, is used. It is approximately 1-1.5 cm thick, open in a circular shape at the center to allow the pulp to flow, and it is designed to sit on the mill outlet throat.

By placing the overflow trunnion on the mill outlet, the mill can be filled with balls up to 45% of its volume. However, having an excess amount of balls in the mill does not necessarily imply better grinding; in fact, an excessive ball load can adversely affect the grinding process. Therefore, mills should not be loaded with balls in excess of 45% of their volume. In many cases, the most optimal grinding environment is achieved when the mill is loaded with balls up to 40% of its volume.

In particular, in compartmentalized mills used for cement grinding, each compartment should be evaluated separately. During the operation of the mill, the operating size of the ball should be added to each compartment, calculated specifically for that compartment.

7.Cylpebs or balls, which one should be preferred?

In mills, grinding occurs at contact points for balls, while for cylpebs, it takes place along the contact lines and surfaces of the cylpebs.


4 balls make contact with each other at 4 different points
4 cylpebs make contact with each other at 2 lines and 2 surfaces

Şekil 5: Bilya ve cylpebs

This implies that in the grinding medium where cylpebs are used, there will be more contact points and surface areas where grinding will take place.

When the grinding unit achieves the desired grinding in the mill, whether it is balls or cylpebs becomes less important. What needs to be considered is the impact of balls or cylpebs on the ore grinding cost per ton.

Cylpebs tend to wear out faster due to having more surface area. On the other hand, the corners of cylpebs will wear each other out more quickly. Since balls are spherical, their wear will be less compared to cylpebs.

Various studies have been conducted on the efficiency of cylpebs and balls in grinding. The most accurate way to determine the difference is to test both grinding media under operating conditions. In a plant where two mills are operated under the same conditions, one should be loaded with balls made from the same material, and the other should be loaded with cylpebs. It is important to note that it will take a certain period depending on the added material and mill volume for all the existing grinding media in both mills to become the newly added grinding media.

For a laboratory comparison, two grinding media should be prepared under the same conditions, and balls and cylpebs should be tested. The tests should be repeated several times, and the ground ore sizes and weight loss in the grinder should be measured. The results should be thoroughly evaluated.

Based on my personal assessment, years ago, in a plant where 2mm-sized magnetite iron ore was ground, $Ø_B$ = 25 mm balls were loaded into one mill, and $Ø_B$ = 25 mm cylpebs were loaded into another mill for observation. After evaluation, it was determined that the cost of cylpebs was slightly lower than that of balls, and cylpebs were subsequently used in the mills.

Due to their round shape, balls move more freely in the mill compared to cylpebs. The ease of movement in the mill enhances the effectiveness of balls in grinding. The advantage of balls over cylpebs in terms of movement decreases as the grinder diameters decrease. As a result, in an environment where cylpebs are used for fine grinding, there will be more contact points than in the case of balls. In the end, I believe that cylpebs are more effective in fine grinding, while balls are more effective in coarse grinding, and the cost difference in grinding will not be significant.

Deciding which is more suitable between cylpebs and balls in grinding requires testing and experimental use of both under operating conditions.

7.Excel Formulas

The ENTERED DATA shown in Table 6 are the required inputs for calculating the grinding ball diameter using the Bond equation.

The data entered between C6 and C14 is being input into the table. While entering these data, attention should be paid to the units as well as the rows and columns.

The calculated values in C17 to C19 represent the required OPERATIONAL SIZE casting/steel ball sizes for grinding in the mills. The size of casting cylpebs, alumina cylpebs, or alumina balls is calculated based on the values in C17 to C19.

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Table 6: Calculations for mill ball diameter and cylpebs size

	A	B	C	D	E
4		CALCULATION OF BALL AND CYLPEBS DIMENSIONS			
5			INPUT DATA		Given data
6		F_{80} =	5.400	mikron	Screen size, 80% of the ore passing
7		W_i =	17,00	kWs/t	Work index
8		N_D =	0,65	$\%N_C$	Mill speed as $\%N_C$
9		ρ_C =	2,85	gr/cm^3	Mineral density
10		$Ø_{Diç}$ =	3,80	metre	İnside diameter of ball mill
11		K_D =	350	Katsayı	Fixed value
12		ρ_D =	7,80	gr/cm^3	Cast iron density
13		ρ_A =	3,70	gr/cm^3	Alumina density
14		ρ_C =	2,60	gr/cm^3	Gravel density
15					
16		Casting ball diameter, $Ø_{DB}$ =	$25.4 * (F_{80} / K_D)^{0.5} * \{(\rho_C * W_i) / [100 * N_D * (3.281 * Ø_D)^{0.5}]\}^{0.33}$		
17		$Ø_{DB}$ =	60	mm	Grinding ball diameter
18		Weight of the cast ball, B_Q =	$4 / 3 * \pi * (Ø_{DB} / 2)^3 * \rho_D$		Grinding ball weight
19		B_Q =	870	gr	Grinding ball weight
20					
21		H =	$Ø_{DS}$		Casting cylpebs
22		S_Q =	$\pi * (Ø_S/2) ^ 2 * Ø_{DS} * \rho_D$	$= B_Q$	Casting cylpebs weight
23		S_Q =	870	gr	Casting cylpebs weight
24		$H=Ø_{DS}$ =	51	mm	Casting cylpebs dimention
25		$H=Ø_{DS} \approx 0,9 * Ø_{DB}$ =	54	mm	Casting cylpebs dimention
26					
27		$Ø_{AB}$			Alumina grindig ball
28		A_Q =	$4/3 * \pi * (Ø_{AB} / 2)^3 * \rho_A$		Alumina grindig ball
29		A_Q =	870	gr	Alumina grindig ball
30		$Ø_{AB}$ =	76	mm	Alumina grindig ball
31					
32		H =	$Ø_{AS}$		Alumina cylpebs
33		A_Q =	$\pi * (Ø_{AS}/2)^2 * Ø_{AS} * \rho_A$		Alumina cylpebs weight
34		A_Q =	870	gr	Alumina cylpebs weight
35		$H=Ø_{AS}$ =	66	mm	Alumina cylpebs dimension
36		$H=Ø_{AS} \approx 0,9 * Ø_{AB}$ =	68	mm	Alumina cylpebs dimension
37					
38		$Ø_{ÇB}$			Grinding gravel
39		$Ç_Q$ =	$4/3 * \pi * (Ø_Ç / 2)^3 * \rho_Ç$		Gravel weight
40		$Ç_Q$ =	870	gr	Gravel weight
41		$Ø_{ÇB}$ =	85	mm	Gravel diameter
42					
43		Doering formulas		Cast iron	
44		$Ø_{DB} = 20.17 * (F_{80}/K_D)^{0.5} * ((\rho_C * W_i)/(100 * N_D * (Ø_D)^{0.5}))^{0.33}$		Bilya	Grinding ball
45		$H=Ø_{DS} = 18.15 * (F_{80}/K_D)^{0.5} * ((\rho_C * W_i)/(100 * N_D * (Ø_D)^{0.5}))^{0.33}$		Silpeps	Grinding cylpebs
46		$Ø_{DB}$ =	58	mm	Grindin ball diameter
47		$Ø_{DS}$ =	52	mm	Cylpebs dimension
48		$H = Ø_{DS} \approx 0.9 * Ø_{DB}$ =	52	mm	Cylpebs dimension
49					

The equations to be used while preparing Table 6 are provided below.

C17=SUM(25,4*(C6/C11)^0,5*((C9*C7)/(100*C8*(3,281*C10)^0,5))^0,33)
C19=SUM((4/3)*3,1416*(C17/2)^3*C12)/1000
C23=SUM(C19)
C24=SUM(((C23*4)/(3,1416*C12))^0,33)*10
C25=SUM(C17*0,9)
C29=SUM(C19)
C30=SUM(((C19/((4/3)*3,1416*C13))^0,33)*10*2)
C34=SUM(C19)
C35=SUM(((C19/(3,1416*2*C13))^0,33)*2*10)
C36=SUM(C30*0,9)
C40=SUM(C19)
C41=SUM((C19/((4/3)*3,1416*C14))^0,33*2*10)
C46=SUM(20,17*(C6/C11)^0,5*((C9*C7)/(100*C8*(C10)^0,5))^0,33)
C47=SUM(18,15*(C6/C11)^0,5*((C9*C7)/(100*C8*(C10)^0,5))^0,33)
C48=SUM(C46*0,9)

The Doering company suggests that if the calculated ball size is below 25 mm, an increase of 20% should be considered, and if the silpebs size is below 22 mm, the grinder size should be taken 30% larger.

After calculating the mill sizes, when the mill is initially loaded, balls should be filled in proportion to the distribution shown in Table 5, and operational size balls/cylpebs should also be added while the mill is in operation.

For efficient grinding in the mill, operational size balls/cylpebs should be added in a timely and continuous manner according to the consumption of balls/cylpebs. This ensures that there are balls/cylpebs of every required size for grinding in the mill. Loading balls/cylpebs of different sizes into the mill during operation is meaningless.

It is important to avoid waiting for the mill to stop grinding to load balls/cylpebs while it is in operation. Instead, the ball consumption in grinding should be determined, and operational size balls should be continuously added to the mill along with the ore, depending on the amount of ore to be ground.

When it is desired to use cast cylpebs instead of cast balls, or especially in the ceramic industry when materials such as alumina balls, alumina cylpebs, or pebbles are preferred over casting, the required grinding sizes are also calculated from Table 6.

Doering International, a manufacturer of balls and cylpebs, provides the equations it uses to calculate the dimensions of cast balls or cylpebs in rows 44 and 45 of the table. These equations represent a simplified form of the Bond equation. There is no significant difference in the calculated dimensions between the results obtained using the Bond equation and these equations.

The calculated operational ball size should be chosen as the next size up between the two ball sizes available in the market, provided that the range between these two values is not too large. On the other hand, when the calculated ball diameters are rounded to the nearest multiple of 5 or 0, there will be no error. For example, if the ball diameter is calculated as Ø = 62 mm, using Ø = 65 mm or Ø = 66 mm would be suitable, and Ø = 70 mm can be chosen when calculated as Ø = 66 mm, as it would still achieve the desired grinding.

Using both balls and cylpebs together in the same mill is not recommended. However, if a decision is made to switch from using balls to cylpebs or vice versa while using balls as the grinding medium in the mill, the desired grinding medium should be added on top of the existing one without emptying the mill. Depending on the mill volume, after a certain period, the grinding medium inside the mill will completely transform into the newly added one. During this period, the ground ore exiting the mill should be closely observed.

In grinding, the essential factor is to maintain the size of the ore to be ground, using a suitable size of grinding medium. If the size distribution of the ore to be ground is variable and unusual, then a grinding medium of a size matching the largest particles in the ore should be selected.

In ore preparation plants, for ball diameter calculations, the unchanged mill diameter $Ø_D$ and the critical speed as a percentage, which is the mill speed N_D, are considered. It should be remembered that as the liner wears, the mill diameter $Ø_D$ may slightly increase. For ore sent to beneficiation plants, it is natural for the values of F_{80}, W_i, and ρ_C to change within certain intervals. If these three values change within small intervals at the plant, the possible effects of this change on grinding should be observed.

Liner wear negatively affects the grinding efficiency of balls. It should be noted that liner wear does not mean thinning of the mill liners but rather the wear of lifters on the liner to the extent that they cannot perform their duties.

Table 7: Densities and Work Indices of Ores

Ore	Density gr/cm^3	Work index, W_i (kwh/st)	Ore	Density gr/cm^3	Work index W_i
Andesite	2.84	22.13	Lead Ore	3.44	11.40
Barite	4.28	6.24	Lead-Zinc Ore	3.37	11.30
Basalt	2.89	20.41	Limestone	2.69	11.61
Bauxite	2.38	9.45	Limestone for Cement	2.68	10.18
Cement Clinker	3.15	13.49	Manganese Ore	3.74	12.46
Cement Raw Material	2.67	10.57	Calcined Magnesite	5.22	16.80
Chrome Ore	4.06	9.60	Mica	2.89	134.50
Clay	2.23	7.10	Molybdenum Ore	2.70	12.97
Gypsum Stone	2.69	8.16	Nickel Ore	3.32	11.88
Coal	1.63	11.37	Oil Shale	1.76	18.10
Bituminous Coal	1.51	20.70	Phosphate Fertilizer	2.65	13.03
Petcoke	1.78	73.80	Phosphate Rock	2.66	10.13
Copper Ore	3.02	13.13	Potash Ore	2.37	8.88
Gold Ore	2.86	14.83	Potash Salt	2.18	8.23
Diorite	2.78	19.40	Pumice	1.96	11.93
Dolomite	2.82	11.31	Pyrite Ore	3.48	8.90
Sand	3.48	58.18	Pyrrhotite Ore	4.04	9.58
Feldspar	2.59	11.67	Quartz	2.64	12.77
Ferrochrome	6.75	8.87	Quartzite	2.71	12.18
Ferromanganese	5.91	7.77	Rutile Ore	2.84	12.12
Ferrosilicon	4.91	12.83	Sandstone	2.68	11.53
Flint	2.65	26.16	Shale	2.58	16.40
Fluorspar	2.98	9.76	Silica	2.71	13.53
Gabbro	2.83	18.45	Silica Sand	2.65	16.46
Galena	5.39	10.19	Silicon Carbide	2.73	26.17
Garnet	3.30	12.37	Silver Ore	2.72	17.30
Glass	2.58	3.08	Sinter	3.00	8.77
Gneiss	2.71	20.13	Slag	2.93	15.76
Granite	2.68	14.39	Blast Furnace Slag	2.39	12.16
Graphite	1.75	45.03	Slate	2.48	13.83
Gravel	2.70	25.17	Sodium Silicate	2.10	13.00
Ilmenite	4.27	13.11	Spodumene Ore	2.75	13.70
Iron Ore	3.96	15.44	Syenite	2.73	14.90
Hematite	3.76	12.68	Kyanite	3.23	18.87
Specular Hematite	3.29	15.40	Tin Ore	3.94	10.81
Oolitic	3.32	11.33	Titanium Ore	4.23	11.88
Limonite	2.53	8.45	Volcanic Rock	2.85	21.10
Magnetite	3.88	10.21	Uranium Ore	2.70	17.93
Taconite	3.52	14.87	Zinc Ore	3.68	12.42